Matrix method for comparing system and individual energy return ratios when considering an energy transition

Carey W. King a, b, 1

Energy Institute, The University of Texas at Austin, 2304 Whitis Ave, C2400, Austin, TX 78712, USA

Center for International Energy and Environmental Policy, Jackson School of Geosciences, The University of Texas at Austin, 2275 Speedway, C9000, Austin, TX 78712, USA

Article info

Article history:
Received 25 February 2014
Received in revised form 8 May 2014
Accepted 10 May 2014
Available online 14 June 2014

Keywords:
Energy
Net energy
Life cycle assessment
Energy economics
Input–output

Abstract

ERRs (Energy return ratios) are valuable metrics for understanding and comparing the contributions of individual energy technologies. It is also important to calculate ERRs in the context of a system, or economy, using a mix of energy technologies. In this paper I demonstrate a framework to simultaneously consider individual energy technology and system-wide ERRs using a process-based input–output model approach. I demonstrate the approach via an example calculating grid electricity ERRs assuming constant technology with only a shift in dominance from fossil to renewable technology. The framework also enables interpretation of changes in individual ERRs due to a shift from one technology to another, with implications for energy scenario analyses. Another finding of this paper is that the ERR GER (gross energy ratio, often assumed equal to EROI (energy return on energy invested at the 'mine mouth')), is only well-defined for primary energy extraction and not energy carriers such as gasoline and electricity. NER (Net energy ratio) and NEER (net external energy ratio), also known as EPR (energy payback ratio), are the most appropriate metrics for describing energy carriers sold to consumers.

© 2014 Elsevier Ltd. All rights reserved.

1. Introduction

The calculation of ERRs (energy return ratios) helps compare the energy and economic benefits of energy technologies and resources. ERRs assess how much energy it takes to produce energy. In the 1970s, researchers established mathematical methods to perform NEA (net energy analysis) to calculate ERRs such as EROI (energy return on (energy) investment) and NER (net energy ratio) [5,6,8,12,13]. These methods considered process LCA (life cycle assessment) information, such as the amount of energy needed to make steel in a foundry, as well as economic information from national accounts. The economic information in the form of I–O (input–output) matrices characterizes the monetary flows among economic sectors per techniques developed by Leontief [5,29,31]. Ref. [5] provides a good example of combining process and I–O information in what is often termed a 'hybrid' analysis that uses both process and economic I–O information. Ref. [5] used process information to estimate flows of energy for the energy sectors of the economy (e.g., oil and gas extraction, coal extraction) while keeping flows in units of money for all other economic sectors.

Despite the mathematical rigor of NEA and LCA, just like models of any system, the outputs are only as good (or bad) as the input information. Garbage in = garbage out. Because of a misunderstanding about what input information is and is not included in NEAs of energy technologies, it is often very difficult to compare the NER for a photovoltaic panel in one paper to the NER for coal electricity in another. This problem is not confined to net energy analyses, as the same problem of comparison occurs when considering similar economic concepts such as LCOE (levelized cost of electricity). Simply stating a calculated value of LCOE for wind and coal-fired electricity does not reveal the assumptions for those calculations, such as discount rate, plant lifetime, quality of wind and coal resources, etc.

By focusing on calculating ERRs using matrix methods, the modeller is forced to consider what information is and is not included in the model. This is particularly important in light of articles that claim to 'clarify' NEA methodology (or really LCA of energy systems in general), but in fact do not create consensus within the research community [46]. Many of the discrepancies among studies relate to differences in definitions of terms used to interpret calculated values as well as the stage of the life cycle at...
which to compare the ERR [3,32,34]. A great amount of effort is required to ‘harmonize’ various LCAs to compare them on equal footing (see Ref. [21] for an example for harmonizing greenhouse gas emissions from LCAs). A sufficient comparison of the literature is beyond the scope of this manuscript as it necessitates its own manuscript itself, as witnessed by articles attempting to do just that [15,18,36,46]. I do summarize in Section 2.2, however, some existing ERR literature and how the ERRs calculated and defined in this manuscript relate to the existing literature.

The explicit writing of input information into matrix forms to structure calculation of ERRs can possibly alleviate confusion among studies, or at least enable clarity of the assumed inputs. In principle, any disagreements should focus on the values to input into the matrix formulations, but not the matrix formulations themselves. The matrix formulations can be of multiple types such as those based upon I–O formulations (as mention previously), the methods of [22] (see Ref. [4]), or perhaps some other organizational system that clearly indicates inputs (energy, materials, money, etc.) needed to calculate the production of some output.

One of the main reasons that matrices are useful organizational structures is that matrix methods force the modeler to input a value of zero for all inputs that are not specifically considered. In many instances the modeler might know that the input value is >0, but the data point might not be available due to lack of knowledge. In other instances, a zero input value correctly means that a given process does not use any input from another process.

In addition, the matrix formulation forces the modeler to consider when he is modeling a given energy input (or embodied energy input) for one component of the model, but not another component. As an example, consider the calculation of NER for electricity from a PV (photovoltaic) array that is connected to the electric grid. The LCA of the PV module might consider the energy input needed to make the aluminum frame of the PV module. The modeler might also like to consider the primary energy (e.g. coal) feedstock into power plants on the grid that could be displaced by the PV electricity [37]. However, the coal-fired power plant, as well as much of the infrastructure (e.g. power lines) composing the electric grid is also composed of aluminum, and many times this material need for all components is not consistent between models. In other words, if an LCA model of PV assumes the existence of a coal-fired power plant without also considering the same input requirements for both coal and PV electricity, then the model is ill-suited for sensitivity analysis. The early energy analyses were generally consistent due calculating embodied energy from the same base of information [5,6]. However, the level of consistency is largely a matter of desired scope, data limitations, or simply researcher interest.

Perhaps a more fundamental discussion is when the modeler assumes some average fuel efficiency of converting primary energy fuels to electricity (e.g. in a coal-fired power plant). For example, approximately 3 Mj of coal are burned for 1 Mj of equivalent electricity. Thus, some researchers assume the EROI of PV electricity can be multiplied by 3 to compare it to a primary energy equivalent of coal. Refs. [37] and [17] call this ‘scaled’ EROI of PV electricity the ‘primary energy equivalent,’ or EROIPV-energy. I address this concept in Section 5.1.

Ref. [32] also discuss the implications for the electric grid power efficiency as it relates to renewables such as hydropower, wind, and solar. These authors note how the IEA (International Energy Agency) counts the energy content of 1 kWh of electricity output from these non-thermal renewables as the engineering equivalent in MJ (e.g., 1 kWh = 3.6 MJ). Given the typical efficiency of steam cycle thermoelectric power systems of ~1/3 [32], states ‘... hydro and wind power appear to make a contribution which is 3 times less than their actual contribution in final energy terms.’

These statements regarding an assumed primary energy equivalent reflect an assumption that renewable energy competes at the margin with the dominant fossil-fueled system. For example, the EIA (Energy Information Administration) of the U.S. Department of Energy does assume that non-thermal power generation (e.g., nuclear, hydropower, wind, PV) has primary energy equivalent based upon the average heat rate of the thermal power generation fleet (e.g. 1 kWh = 10 MJ). However, this assumption of a primary energy equivalent is not universally accepted and does not help envision a world free of fossil fuels because it inherently assumes their existence. In short, the EIA and IEA, two of the most important sources for energy data, do not agree on how to count the primary energy of electricity originating from non-combustible resources. Thus, the discussion of the primary energy equivalent of non-combustible renewable electricity is beyond that of net energy analysis.

How can we imagine a fossil fuel free world if the definition of renewable energy assumes the existence of and/or dependence upon combustible fuels?

In this manuscript I specifically do not make the assumption of a thermal primary equivalent for non-thermal renewable electricity because the model itself does not specifically include any information on marginal energy consumption. There is no need to assume primary energy equivalents for renewables as defined by fossil fuel (or other heat-based) electricity technologies. Generally, only humans are concerned about marginal versus absolute impacts. Further, the thermal-equivalent assumption confuses the issue of calculating all primary energy resource inputs including insolation. In this paper I demonstrate both how to consider the grid efficiency in the LCA model itself and how one can just as easily choose solar energy as the numeraire metric for describing the efficiency of the grid versus combustible feedstocks such as coal.

The goals and organization of this paper are as follows:

- Section 2 describes the methods that use a linear equation framework with process LCA input information using terminology and structure of the energy analysis approaches using the input-output Leontief structure. To provide some context of this work compared to a vast existing literature, Section 2.2 compares the ERR formulations in this paper to a subset of the literature. I also reiterate general modeling guidelines in some of the literature.
- Section 3 describes an example problem formulation that demonstrates calculation of system-wide and individual ERRs when transitioning from 99% fossil to 99% renewable electricity. By defining ERRs for fossil, renewable, and the grid mix I demonstrate the relationships among them assuming constant technology.
- Section 4 describes example results.
- Section 5 discusses interpretation of the results in terms of coherently using LCA models to conceptualize an energy transition.

2 Material and methods

Equation (1) shows the structure of the energy analysis I–O (input–output) method where each of n economic sectors (or processes) are assumed to be in energy balance (see Refs. [5,6,8]). EMearth can generally be an n × n matrix with m ≤ n primary energy resources extracted from the Earth. Thus, there are n − m rows having all zeros such that the m primary energy resources are represented by m of the rows. X is an n × n diagonal matrix of total gross output, Xj, of each economic sector (or process) on the

\[ E_{\text{Earth}} = X \times X \]

\[ X_{\text{Earth}} = \begin{pmatrix} E_{\text{coal}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & E_{\text{coal}} \end{pmatrix} \]

\[ X = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix} \]
diagonal. Equation (1) is solved for a \( n \times n \) matrix of energy intensities, \( \varepsilon \) (see Equation (2)), that characterize what is termed the ‘embodied energy’ of products from each economic sector (or process). The only non-zero rows of \( \varepsilon \) correspond to rows with non-zero energy inputs of \( E_{\text{earth}} \).

\[
\varepsilon X + E_{\text{earth}} = \varepsilon X
\]

\[
\varepsilon = E_{\text{earth}}(X - \bar{X})^{-1}
\]

In most basic terms, these equations represent a set of linear equations describing each \( j \)th process or economic sector as a column that ‘buys’ units of input from each given \( i \)th row to produce its output.

Equation (2) can be reformulated in terms of a normalized matrix of technological coefficients \( A \) and where \( \varepsilon = E_{\text{earth}}\bar{X}^{-1} \) is a matrix the same size as \( E_{\text{earth}} \) where there is a 1 in the location that energy is input into each energy extraction sector and 0 otherwise for all other sectors (or processes). Here matrix \( A \) is a normalized version of \( X \) but not necessarily as typical for economic transactions or use matrices since \( X \) can have mixed units.

\[
\varepsilon = E_{\text{earth}}\bar{X}^{-1}(I - A)^{-1} = \varepsilon(I - A)^{-1}
\]

Each \( X_{ij} \) is generically in units of ‘gross energy input’ divided by net ‘something output’. Each ‘gross energy input’ refers to those units of rows in \( E_{\text{earth}} \), and each ‘something’ is dictated by the units of the rows in the transactions matrix, \( X \). Using traditional economic transactions I–O matrices, each row is composed of monetary values as taken from governmental statistics of the value added passed as inputs from one economic sector to another \[12,13\]. For these I–O matrices of national accounts with elements \( X_{ij} \) in units of money, a ‘conservation’ relationship constrains \( \sum_{i} X_{ij} = \sum_{j} X_{ij} \) where each \( X_{ij} \) is divided by the sum of all rows \( i \) for each column \( j \) \[31\]. Thus, \( A_{ij} = \frac{X_{ij}}{\sum_{j} X_{ij}} \). With mixed units in \( X \), this conservation of flow concept is not applicable.

### 2.1. Energy return ratio formulas

For this paper, I assume process based (bottom-up) information with which to calculate various ERRs (energy return ratios). This is similar to that in Ref. [4] that use the LCA structure of [22]. I calculate the ERRs as a function of \( \varepsilon \) by modeling each process as a different column in a transactions matrix nominally composed of direct energy consumption for each process.

A high level form of \( \text{GER} \) (gross energy ratio) and \( \text{NER} \) (net energy ratio) are shown in Equations (4) and (5) (also see Ref. [14]), and depending upon what net energy question is of interest, the \( \varepsilon \) can be a summation of several \( \varepsilon_i \)’s. It is important to note, that ERR equations as a function of \( \varepsilon_{ij} \) work only for energy sectors (or processes) that model output energy carriers in units of energy since each \( \varepsilon_{ij} \) must be in units of ‘gross energy input’ per ‘net output energy’ (e.g., not per ‘net output money’, ‘net output mass’, etc.). I use the terms \( \text{GER} \) and \( \text{NER} \) from Refs. [3] and [4] because the names themselves help the modeler focus on the energy source and/or product of interest.

One can use a ‘total’ \( \varepsilon \) mathematically by summing all \( i \) rows for a given column \( j \) of \( \varepsilon \) in Equation (2), and this describes the total primary energy input for that \( j \)th output. The factor ‘\(-1\)’ in the denominators of Equations (4) and (5) subtracts the net output of the energy sector(s) of interest from its gross extraction so that the denominator sums to only the total intermediate energy allocated within the system to produce net output energy.

\[
\text{GER} = \frac{\varepsilon_{\text{ROIMm}}}{\varepsilon_{\text{Intermediate Energy Demand}}} = \frac{\varepsilon_{\text{Gross energy}} - \varepsilon_{\text{Net energy}}}{\varepsilon_{\text{Gross energy}}} = \frac{\varepsilon_{\text{Gross energy}} - \varepsilon_{\text{Net energy}}}{\varepsilon_{\text{Gross energy}}} - 1 = \frac{\varepsilon}{\varepsilon - 1}
\]

\[
\text{NER} = \frac{\varepsilon_{\text{Intermediate Energy Demand}}}{\varepsilon_{\text{Net energy}}} = \frac{\varepsilon_{\text{Gross energy}} - \varepsilon_{\text{Net energy}}}{\varepsilon_{\text{Net energy}}} - 1 = \frac{1}{\varepsilon - 1}
\]

Consider all processes included in \( X \) or \( A \). The \( \text{GER} \) and \( \text{NER} \) calculated using Equations (4) and (5), respectively, are related as in Equation (6). Thus, Equation (6) is a check on the mathematics of calculating system-wide \( \text{GER} \) and \( \text{NER} \). Further, it can be shown that the minimum \( \text{GER} \) is unity and minimum \( \text{NER} \) is zero. At these minimum values, all \( \varepsilon_{ij} = \infty \). An important implication of this mathematical fact is that if one models all primary energy supplies within \( X \) or \( A \), they all produce net energy if any output is positive \( \varepsilon_{ij} > 0 \), or none produce net energy if all outputs are zero \( \varepsilon_{ij} = 0 \).

\[
\begin{align*}
\text{GER} & = \frac{\varepsilon}{\varepsilon - 1} \\
\text{GER} - 1 & = \frac{\varepsilon - 1}{\varepsilon - 1} - \frac{\varepsilon - 1}{\varepsilon - 1} \\
\text{GER} - 1 & = \frac{1}{\varepsilon - 1} \\
\text{GER} & = \frac{\varepsilon}{\varepsilon - 1}
\end{align*}
\]

The calculations of \( \text{GER} \) and \( \text{NER} \) relative to the input of each \( i \)th primary energy resource are shown in Equations (7) and (8), respectively. The \( \text{NER} \) for final product energy carriers (e.g., electricity) is calculated using Equation (9), a slight variation Equation (8). Energy carriers are not primary energy resources, by definition, because some process or consumption of primary energy has occurred to transform primary energy to an energy carrier. Energy carriers can be a combination of multiple technologies (e.g., the electric grid), and thus one can calculate \( \text{NER} \) (and other ERRs) of electricity (and other energy carriers) relative to a particular primary energy resource (see Equation (9)) where \( i \) represents the sector that extracts the primary energy.

\[
\begin{align*}
\text{GER}_{\text{primary}, i} & = \frac{\text{Total gross extraction of } i \text{th primary energy}}{\text{Intermediate consumption of } i \text{th primary energy}} \\
\text{GER}_{\text{primary}, i} & = \frac{\varepsilon_{ij}}{\varepsilon_{i,j} - 1}
\end{align*}
\]
\[\text{NER}_{\text{primary},i} = \frac{\text{Total net extraction of } i\text{-th primary energy}}{\text{Intermediate consumption of } i\text{-th primary energy}} = \frac{1}{\epsilon_{i,j} - 1} \quad (8)\]

\[\text{NER}_{\text{carrier},ij} = \frac{\text{Total net output of } j\text{-th energy carrier}}{\text{Intermediate consumption of } i\text{-th primary energy}} = \frac{1}{\epsilon_{i,j} - 1} \quad (9)\]

The system wide GER (of primary energy resources) and NER (of primary energy resources or energy carriers) across all primary energy types (considered simultaneously) uses the same structure as before except one must sum across the rows of \( \epsilon \) to create a single vector of energy intensities (see Equations (10) and (11)). The term \( \left( \sum_{i,j} \epsilon_{i,j} Y \right) \) in Equations (10) and (11) are what many authors term the cumulative energy demand, or CED [32,37], that is all primary energy consumed for intermediate inputs and in producing output energy carriers (i.e., CED includes feedstocks required to produce energy carrier outputs).

\[\text{NEER}_{\text{system}} = \frac{\text{Total net output of energy carriers}}{\text{Intermediate consumption PE} - \text{direct PE input as fuel}} = \frac{\sum_{i} Y_i}{\left( \sum_{i} \epsilon_{i,j} \right) Y - \sum Y} \quad (12)\]

Just as one can calculate GER and NER system values as well as GER and NER values specific to only one type of primary energy, one can do the same for NEER (see Equation (13)).

\[\text{NEER}_{\text{primary},j} = \frac{1 \text{ unit of } j\text{-th energy output}}{\text{Total intermediate consumption of } j\text{-th PE} - \text{direct } j\text{-th PE input as fuel}} = \frac{1}{\epsilon_{i,j} - \text{direct } j\text{-th PE input as fuel}} \quad (13)\]

2.2. Relating ERRs of this paper to the literature

Here I briefly summarize how the GER, NER, and NEER here relate to the net energy literature. This paper does not have a goal of reconciling all ERRs in the literature, but I provide a brief comparison. For some useful background and summaries, I refer the reader to [20,32,34]; and [4]. Specifically both [32] and [34] do a good job of comparing various ERR metrics from the literature, and I relate my calculations in the light of discussions of those papers as well as a few others. What this manuscript does add to the literature is the use of the I–O framework to conceptualize changes in individual and system-wide (e.g., electric grid) ERRs during an energy transition.

Ref. [34] uses much information from Ref. [33] and emphasizes that comparing ERRs relies on understanding the assumptions regarding (1) boundaries of the system under analysis, (2) energy quality corrections, (3) energy-economic conversions, and (4) the sources and comprehensiveness of the underlying data inputs. The question of the boundary of each net energy analysis is perhaps the primary reason for difficulty in comparing ERR metrics across the literature. Ref. [34] describes that the analysis of boundaries can...
occur across two different ‘dimensions’ (see their Table 1). One dimension describes the calculation of the numerator of ERRs in terms of an expanding boundary across the process, or supply, chain from extraction to processing to distribution (e.g., ‘What are the energy outputs?’). The second dimension describes the calculation of the denominator of ERRs as expanding boundaries, or levels, of inputs (Levels 1–5 in Ref. [34]): energy input from the primary energy supply (or supplies) under investigation, energy input from other supply chains, energy embodied in materials, energy embodied with labor, and energy embodied in various other economic services (e.g., financial and legal services).

The difficulty in comparing ERR results is that it is generally possible to incompletely consider energy inputs across even the most broad boundary for analysis. That is to say if there are five inputs to consider at each input boundary level, but the modeler only inputs three of the five at each level, then there are still missing inputs even though the most expansive boundary has been considered. For this reason, it can be very useful to plot or list the ERR calculation at different levels of the analysis from beginning to end (see Table 3 and Figure 8 of Ref. [23]).

This issue of considering all inputs is similar for economic analyses (e.g., calculating levelized cost of electricity) but perhaps not perceived as much of a difficulty to include all costs in a cash flow because cost information (e.g., labor costs) is more prevalent in monetary units though still with uncertainty. To consider indirect and embodied energy inputs for an ERR one must often rely on economic–O analyses to convert monetary costs to energy units [6].

Ref. [34] also suggest the use of quality-corrected energy inputs and outputs in calculating ERRs. The quality corrections are primarily meant to incorporate qualities of primary energy resources and energy carriers that are not characterized by energy content alone (e.g., energy per kg). Several papers have characterized or used different quality-corrected energy flows that are most commonly weighted by energy commodity prices in some manner [9,11,34,47,48]. I do not address quality-corrected flows in this manuscript.

The GER of Equation (4) is mathematically equivalent to energy return on (energy) investment of extracted primary energy ‘at the mine mouth’ (EROImm) or EROI at extraction as in Ref. [34]. Conceptually, I consider GER applicable to describe the stage of primary energy extraction. Ref. [4] use GER more loosely to describe energy carriers as well as primary energy, and do not explicitly conceptualize the relation of GER = NER + 1 (Equation (6)). To my knowledge, this is the only manuscript to recognize this constraint on GER and NER (as defined here).

The NER of Equation (5) is not identical to the NER in Refs. [32,38,39]. The previous authors calculate NER with a denominator including 100% of the energy content of primary energy feedstock into a refining or conversion process (e.g., fuel to electricity in a power plant). My formulation here excludes the energy in the exported electricity from the denominator because that energy content is not consumed within the system defined by the matrices A and X.

The NEER of Equation (12) is the same as the EPR (energy payback ratio) in Refs. [32]; the EER (external energy ratio) in Refs. [38,39]; net external energy ratio in Refs. [3,4]; and EROI of an energy carrier (EROI of [34]). While Ref. [32] refers to EROI as calculating the ‘Primary energy product compared to primary energy invested in upstream activities’., their mathematical definition is more generally that of NEER calculated for an intermediate product in the life cycle (e.g., coal delivered to a power plant that occurs after coal is extracted at the mine but before coal is burned for electricity).

EROI (or EROE) is perhaps the most common and loosely-used ERR term, and one can consider the ‘EROI of this or that’ equivalent to discussing the monetary ‘cost of this or that.’ Specifying the point in the energy supply chain or life cycle at which to quote the EROI, or cost, that you are discussing inherently defines the boundary of ‘at the mine mouth’ (EROImm), ‘delivered electricity’ EROIelec, etc.

As I show in this paper, GER = EROImm is only well-defined at the first step in the life cycle at which the primary energy is available. In this sense, GER = EROImm only makes sense for gross quantity of primary energy before it is ever delivered as or converted to an intermediate product, energy carrier, or final energy product. In other words, there is no such thing as either gross extracted intermediate products or final products extracted from the Earth, such that the definition of GER = EROImm is inappropriate for all but the point of primary energy extraction. The EROI of delivered energy carriers, such as electricity as EROIelec, is equivalent to NEER. Generally EROI is not calculated using the primary energy content of feedstocks as energy input in the denominator of ERRs. The reason GER = EROImm = EROI and NEER = EROIcarrier = EROI (numbers referring to Table 1 of Ref. [34]) is that at the beginning of the energy supply chain (before any refining or conversion) there is not yet a defined feedstock. To convert primary energy to an energy carrier a feedstock must then be defined. This discussion does not mean that one cannot mathematically calculate GER for an energy carrier, only that it has less physical meaning.

There is little evidence for high-level energy policy making use of ERRs. The United States government referred to the use of net energy analysis in the NonNuclear Energy Research and Development Act of 1974, but no energy policies were clearly driven by net energy analysis. After the Energy Policy Act of 2005 led to a mandate for biofuels consumption in the U.S., a series of papers analyzed net energy of corn-based ethanol establishing the low value of ERRs, but they were largely deemed to break some critical value [15]. The analyses, however, usually focused on the ethanol processes only, without simultaneously modeling other energy carriers. Granted, the data challenges can be large, but as discussed in Section 2.1, as long as there is any modeled output, each ERR for each primary energy supply is greater than its mathematical minimum. That is to say, net energy is produced as long as there is any output, and it is practically impossible to have no output for an open system. This leads to the important research question of determining the practical critical ERR values for energy carriers since these are not the mathematical minimum values.

2.3. Steps for net energy analysis

The steps for an analysis of ERRs is similar to general modeling practices in terms of defining the goals of the analysis, stating system boundaries and assumptions, and identifying inputs and outputs. I have nothing to add beyond the existing literature, and I refer the reader to [5,6,8,34] for modeling steps.

3. Calculations

Consider a simplified model of electricity generation with seven processes: (1) primary fossil energy extraction, (2) fossil energy conversion to electricity, (3) renewable energy to electricity, (4) grid mix electricity (some combination of fossil and renewable electricity), (5) materials extraction and processing, (6) primary renewable energy extraction, and (7) renewable electricity storage. Assume the technical coefficients matrix as in Equation (14). The numbers used here are not meant to be exact representations of the real ERRs of fossil energy or any form of electricity, either in the United States or any other location. However, they reside in the
range of values calculated in the literature for oil to electricity and photovoltaic electricity [4].

Fig. 1 is a diagram indicating all non-zero modeled energy flows. The output vector, $Y$, is of size $n/C^2$ to represent output from any of the $n = 7$ modeled processes, but my example considers only non-zero electricity output from fossil electricity, renewable electricity, or grid electricity ($n = 2, 3, 4$). All units in $A$ are input (row) per unit output (column) per a unit of time, but for clarity time is not shown in the units (see Appendix for units of each element in $A$).

$$A = \begin{bmatrix}
0 & A_{1,2} & 0.1 & 0 & 0.1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & (1 - \alpha)(1 + \beta) & \phi & 0 & 0 \\
0.02 & 0 & 0.1 & 0 & 0.1 & 0 & 0.02 \\
0.05 & 0.1 & 0.2 & 0.1 & 0.1 & 0 & 0.056 \\
0 & 0 & A_{6,3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & (1 - \alpha)(1 + \beta)(1 - \phi) & \sqrt{\eta_{stor}} & 0 & 0
\end{bmatrix}$$

Fig. 1. This energy flow diagram indicates the energy flows corresponding to the non-zero inputs in Equation (14) showing the technical coefficient $A$ matrix.

$$A_{1,2} = \frac{1}{\text{Fossil Electricity Efficiency}} = \frac{1}{\eta_{fossil}}$$ (15)

$$A_{6,3} = \frac{1}{\text{Renewable Electricity Efficiency}} = \frac{1}{\eta_{renew}}$$ (16)

where each $\eta$ is a conversion of feedstock energy content to electricity energy content, $\alpha$ is the fraction of grid electricity that is supplied by fossil electricity (where $(1 - \alpha)$ is the fraction of the grid from renewable electricity), $\beta$ is the fraction of generated

In the $A$ matrix, the following physical factors are relevant for this analysis:

- Fossil energy extraction (Fossil J/output)
- Fossil electricity (Fossil elec. J/output)
- Renewable electricity (Renew elec. J/output)
- Grid electricity (Grid elec. J/output)
- Materials extraction and processing (kg/output)
- Renewable energy extraction (Renew J/output)
- Storage electricity (Stored elec. J/output)

electricity lost (Coulomb losses) during transmission and distribution on the grid, $\phi$ is the fraction of renewable electricity sent directly to the grid (equal to capacity factor of the renewable
electricity generation per [7], and \( \eta_{stor} \) is the round-trip efficiency of the electricity storage technology.

This paper assumes \( \eta_{renew} = 0.15 \) as the conversion of feedstock insolation energy to electricity output via a PV (photovoltaic) panel, \( \varphi = 0.12 \) for PV capacity factor, \( \eta_{fossil} = 1/3 \) for a fossil-fueled power plant, \( \eta_{stor} = 0.9 \) for Li-ion batteries as a storage technology. The renewable electricity storage technology is assumed to need some small amount of grid electricity and materials during charging. The storage technology is assumed to recover the renewable electricity \((\mathcal{A}_2^3 = \sqrt[3]{(1 - \alpha)(1 + \beta)/\eta_{stor}})\) before delivering it back to the grid. For a discussion of EROI when sizing storage for making a dispatchable renewable electricity and storage combination, see Ref. [7]. For simplicity, I assume no storage of fossil electricity \((\mathcal{A}_2^3 = 0)\).

For my example, all electricity generation destined for output first flows through the grid. For reference to the choice of \( \beta = 6\% \) used in this paper, the T&D (transmission and distribution) providers within the ERCOT (Electric Reliability Council of Texas) report approximately 6.3% for T&D losses [45]. The amount of total electricity generation before transmission is greater than the demand (or net output) of grid electricity such that 1 unit of grid electricity output is associated with the following quantity of electricity generation:

\[
\mathcal{A}_{2,4}^3 + \mathcal{A}_{3,4}^3 + \mathcal{A}_{2,3,7}^3 = (1 + \beta) [\alpha + (1 - \alpha)\varphi + (1 - \alpha)(1 - \beta)/\eta_{stor}].
\]

I model both the fossil energy stock input and the renewable energy flow input. The fossil energy extracted from the Earth is \( E_{1,1} \) by the fossil extraction process, and the renewable energy flow extracted from the Earth is \( E_{0,6} \) as an input to the renewable electricity process. I explicitly model renewable energy input since for a technology to have renewable electricity output, it must have renewable primary energy flow as an input. Equation (17) shows where the primary fossil energy stock and renewable energy flow enter the modeled system. For an assumed output demand \( Y \) of end products, one can solve for the total extraction of primary fossil energy \( E_{1,1} = E_1 = \varepsilon_1 Y \) and primary renewable energy \( E_{0,6} = E_0 = \varepsilon_0 Y \).

\[
E_{\text{Earth}} = \begin{bmatrix}
E_{1,1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & E_{0,6} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(17)

Table 1

<table>
<thead>
<tr>
<th>% Grid that is fossil electricity ( a )</th>
<th>GERsys fossil energy</th>
<th>GERsys renewable energy</th>
<th>NERsys fossil electricity</th>
<th>NERsys renewable electricity</th>
<th>NERsys grid electricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>11.1</td>
<td>( \infty )</td>
<td>0.43</td>
<td>0.16</td>
<td>0.38</td>
</tr>
<tr>
<td>50%</td>
<td>6.84</td>
<td>( \infty )</td>
<td>0.38</td>
<td>0.15</td>
<td>0.18</td>
</tr>
<tr>
<td>10%</td>
<td>5.27</td>
<td>( \infty )</td>
<td>0.36</td>
<td>0.15</td>
<td>0.13</td>
</tr>
<tr>
<td>1%</td>
<td>5.02</td>
<td>( \infty )</td>
<td>0.35</td>
<td>0.15</td>
<td>0.12</td>
</tr>
</tbody>
</table>

4. Results

In my example model, all three types of electricity (fossil, renewable, and grid) are inherently assumed to have exactly the same qualities to the consumer because nothing is modeled to make any distinction. There is no assumption about how temporal demand for electricity (e.g., high demand during the day and low demand at night) can affect the calculation. Thus, inherently my example assumes the ERRs are the same no matter if the demand is constant or not over an entire day. Also, in the example model here there is no property concerning environmental attributes that might cause a consumer to be more or less favorable to fossil or renewable electricity. See the Appendix for a slightly different assumption on modeling the grid mix with no renewable electricity storage where the fossil technology has a declining efficiency as a function of the fraction of grid electricity supplied by renewables.

In the results of this section there is only variable changing in the technical coefficient matrix \( A \) is \( \alpha \), the percentage of grid electricity from fossil electricity. From 99% fossil energy \((\alpha = 0.99)\) to 99% renewable energy \((\alpha = 0.01)\), Table 1 indicates the GER (or EROI_{sys}) for fossil energy extraction and NER for each of the three forms of electricity as we change the mix of the grid electricity. For each \( \alpha \), there is a different amount of primary fossil energy \((T1)\) and primary renewable energy \((T3)\) extracted as necessary inputs to produce \((net)\) output electricity.

Due to the assumptions of matrix \( A \), as the percentage of fossil electricity in the grid decreases, all ERRs decrease, including the GER_{system, fossil} of primary fossil energy, except for the GER_{system, renewable} of primary renewable energy. Because renewable energy is assumed to flow on the Earth without human intervention, its GER is equal to infinity in all situations.

Note in Table 1 that the system NER_{sys} for fossil electricity is greater than its conversion efficiency \((\text{NER}_{sys, fossil energy} > \text{NER}_{fossil} = 1/3)\). This might appear to be a mistake in that one might think of NER as specifically \( \eta_{fossil} = 1/3 \) because for 1 unit of electricity generation one needs 3 units of primary fossil energy. Upon closer inspection of what is being modeled consider that the NER definition assumes only 2 of those units of primary fossil energy have been dissipated and the other unit has been converted to electricity as output, not yet dissipated as heat. For example [39], and similarly Ref. [32] defines NER with a divisor of ‘\( \text{Eff} = \text{fossil fuel consumed within the system} \) (here consumed = dissipated as heat) and end up calculating NER_{elec} < \eta.

As I have just noted, the system is defined by matrices \( A \) and \( X \), and to have a unit of fossil electricity net output \((e.g., Y_{2,0} > 0 \text{ and/or} Y_{4,0} > 0 \text{ with} \alpha > 0)\), the system itself cannot internally dissipate all the energy content of the fossil energy feedstock. In the case of a coal-fired power plant, it converts 100% of the coal mass primarily to ash and CO2 plus H2O during combustion \((e.g., \text{conservation of mass})\), and it converts 100% of the coal energy content to both heat and electricity. The power plant, however, does not dissipate as heat \((e.g., 'consume') \text{ 100% of the coal energy content because it exports some of it as electricity. The upper limit of NER assuming only the electricity conversion efficiency of a power plant is not } 1/\eta, \text{ but instead is NER}_{\text{upper limit, electricity conversion}} = 1/\eta - 1).
Table 2
The total quantity of fossil energy extraction depends on the grid mix of electricity. Each of columns 3–5 assume 1 unit of electricity net output from only one type of electricity (fossil, renewable, or grid).

<table>
<thead>
<tr>
<th>% Grid that is fossil electricity (α)</th>
<th>GER_{fossil} fossil energy</th>
<th>Fossil energy extraction for net fossil electricity</th>
<th>Fossil energy extraction for net renewable electricity</th>
<th>Fossil energy extraction for net grid electricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>11.1</td>
<td>3.34</td>
<td>0.52</td>
<td>3.57</td>
</tr>
<tr>
<td>50%</td>
<td>6.84</td>
<td>2.20</td>
<td>0.33</td>
<td>1.95</td>
</tr>
<tr>
<td>10%</td>
<td>5.27</td>
<td>2.08</td>
<td>0.17</td>
<td>0.54</td>
</tr>
<tr>
<td>1%</td>
<td>3.02</td>
<td>1.04</td>
<td>0.14</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Table 3
The total quantity of renewable energy extraction depends on the grid mix of electricity. Each of columns 3–5 assume 1 unit of electricity net output from only one type of electricity (fossil, renewable, grid).

<table>
<thead>
<tr>
<th>% Grid that is fossil electricity (α)</th>
<th>GER_{renew} renewable energy</th>
<th>Renewable energy extraction for net fossil elec</th>
<th>Renewable energy extraction for net renewable elec</th>
<th>Renewable energy extraction for net grid elec</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>∞</td>
<td>0.01</td>
<td>6.68</td>
<td>0.09</td>
</tr>
<tr>
<td>50%</td>
<td>∞</td>
<td>0.40</td>
<td>7.18</td>
<td>4.50</td>
</tr>
<tr>
<td>10%</td>
<td>∞</td>
<td>0.74</td>
<td>7.62</td>
<td>8.35</td>
</tr>
<tr>
<td>1%</td>
<td>∞</td>
<td>0.82</td>
<td>7.72</td>
<td>9.24</td>
</tr>
</tbody>
</table>

I now discuss results for NEER. Consider that if there is no fossil electricity net output, there is no need for the additional 3 × units of primary fossil energy input as in A_{1.2}. Thus, in calculating the NEER of fossil electricity, I subtract the necessary 3 × units of fossil energy feedstock that directly relate to the system output, as indicated in Equation (13). This same concept holds for primary renewable energy feedstock. For the example of this paper, if producing 1 unit of fossil electricity, I subtract the necessary 3 × units of fossil electricity net output, there is no need for the additional 3 × units of fossil primary energy input as in Equation (19).

\[
\text{NEER}_{\text{fossil}} = \frac{1}{\epsilon_{1.2} - A_{1.2}(Y_2 + A_{2.4}Y_4)}
\]

\[
\text{NEER}_{\text{renew}} = \frac{1}{\epsilon_{6.3} - A_{6.3}(Y_3 + (A_{3.4} + A_{7.4})Y_4)}
\]

As an example calculation for system wide NEERsystem, assume there is only 1 unit of grid electricity demand, Y_1 = 0 except for Y_4 = 1, and 50% of grid electricity is fossil electricity (e.g., α = 0.5). This example NEERsystem, grid is calculated in Equation (20). Table 4 shows the NEERsystem,j of each type of electricity output as the percentage of fossil electricity of the grid varies.

<table>
<thead>
<tr>
<th>% Grid that is fossil electricity (α)</th>
<th>NEER_{fossil} fossil electricity</th>
<th>NEER_{renew} renewable electricity</th>
<th>NEER_{system} grid electricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>2.85</td>
<td>1.89</td>
<td>2.33</td>
</tr>
<tr>
<td>50%</td>
<td>1.67</td>
<td>1.18</td>
<td>1.02</td>
</tr>
<tr>
<td>10%</td>
<td>1.23</td>
<td>0.89</td>
<td>0.63</td>
</tr>
<tr>
<td>1%</td>
<td>1.16</td>
<td>0.84</td>
<td>0.57</td>
</tr>
</tbody>
</table>

5. Discussion
One important point from this analysis is that it mathematically shows, using the constructs of life cycle assessment and input–output methods, what many energy-economics authors have stated: there is no primary energy resource that serves as an absolute basis for energy quality comparisons. Comparing the joules of energy in sunlight to joules of energy in oil depends upon the technologies that convert each of them to energy services (e.g., heat, light, power, transport) [16]. However, when comparing pathways for the same energy service (e.g., electricity), some net energy literature discusses how one might assume a comparison of renewable electricity (an energy carrier) to fossil primary energy. I now comment on this assumption.

5.1. Using the model to consider ‘primary energy equivalents’

The notion of choosing a fundamental energy numeraire has been explored by various biophysical and other economists. Howard T. Odum explored the concept of ‘energy’ as embodied solar energy (over all time) as a unit energy basis [35]. Several
authors including Cleveland, Hall, Stern, Kaufmann, and Zarnikau compared various ways of ‘aggregating’ energy into a single equivalent metric or basis using price information. These aggregation methods include the Divisia Index for aggregation as well as relative energy prices (e.g., price of electricity relative to coal) to compare other primary energy and energy carriers [9–11,34,47,48].

Table 5
A comparison of ‘fossil primary energy equivalent’ of the NEER (EROI) of renewable electricity to the GER of fossil primary energy shows how both change as the electricity grid mix changes.

<table>
<thead>
<tr>
<th>% Grid that is fossil electricity (a)</th>
<th>NEERsys, renewable electricity</th>
<th>NEERsys, renewable electricity fossil PE equivalent</th>
<th>GERsys, fossil energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>1.89</td>
<td>5.67</td>
<td>11.1</td>
</tr>
<tr>
<td>50%</td>
<td>1.18</td>
<td>3.54</td>
<td>6.84</td>
</tr>
<tr>
<td>10%</td>
<td>0.89</td>
<td>2.67</td>
<td>5.27</td>
</tr>
<tr>
<td>1%</td>
<td>0.84</td>
<td>2.52</td>
<td>5.02</td>
</tr>
</tbody>
</table>

Table 6
The ‘renewable primary energy equivalent’ of the NEER (EROI) of fossil electricity changes as the electricity grid mix changes, but the GER of renewable primary energy is always infinite.

<table>
<thead>
<tr>
<th>% Grid that is fossil electricity (a)</th>
<th>NEERsys, fossil electricity</th>
<th>NEERsys, fossil electricity renewable PE equivalent</th>
<th>GERsys, renewable energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>2.85</td>
<td>19.0</td>
<td>∞</td>
</tr>
<tr>
<td>50%</td>
<td>1.67</td>
<td>11.1</td>
<td>∞</td>
</tr>
<tr>
<td>10%</td>
<td>1.23</td>
<td>8.20</td>
<td>∞</td>
</tr>
<tr>
<td>1%</td>
<td>1.16</td>
<td>7.73</td>
<td>∞</td>
</tr>
</tbody>
</table>

The 3rd column of Table 4 can be scaled by 1/t foss = 3. The NEERsys, renewable elec. = 0.84 with α = 1% becomes NEERsys, fossil-eq = 2.52. The claim is that this can be compared to GERfossil = 5.02 as in Table 1 (same concept as EROI of [37]). Table 5 shows the NEER of renewable electricity scaled to fossil primary equivalents as compared to GER of fossil energy.

Similarly, the NEER of fossil electricity in the 2nd column could be multiplied by 1/t renewable = 6.67 to approximate the primary renewable energy equivalent of net fossil electricity (see Table 6).

For example, NEERsys, fossil elec. = 2.85 with α = 99% becomes GERsys, fossil elec., renewable-eq = 19 scaled to ‘renewable equivalent.’ This value would then ‘compare’ to GERrenew = ∞ (e.g., sunlight is free). Thus, even if scaling up the NEERsys, fossil elec. to its primary renewable energy equivalent, the ‘renewable equivalent’ scaled value will always be smaller than GERrenew!

There is no answer as to which is a more correct interpretation: fossil energy carriers as renewable primary energy equivalents or renewable energy carriers as fossil primary equivalents.

5.2. Focus on extraction of each primary energy flow and stock

As an additional thought experiment, consider if, instead of being a human on the surface of the Earth, you were a subterranean being that lived in a coal seam. In this alternative ‘upside down’ world, GERsunlight = ∞ as it is exposed to you at no effort, but GERfossil is finite because it takes effort to ‘drill’ up to the sunlight at the Earth surface! Instead of comparing one carrier that uses one primary energy feedstock to a second energy carrier that uses a different primary energy feedstock, it is much easier just to state how much of each primary energy resource type is needed for a given desired output (Tables 2 and 3). At that point, one can focus on the nuances of the difference between the energy resources (e.g., temporal implications of using primary energy stocks versus flows) that govern their ability to provide equivalent energy services. LCA models don’t independently inform the ability to substitute one primary energy resource for another; the inputs required for equivalent substitution must be known from engineering and physical system modeling as inputs into the LCA model.

5.3. Seeing the forest instead of the trees: the need for relating ERRs to the economy

Why do we need ERR metrics at all? Many claim we can simply use economic cost metrics and ignore energy-based metrics such as ERRs. In this case, those of us calculating ERRs are only a small group of analysts separated from the decisions of the real world. I do believe there is a role for ERRs in long-term thinking and decision making, and there is a great need to translate among ERR, LCOE, and other economic and cost calculations. This translation is one way to interpret the relevance of ERRs to the non-academic world and better explain the role of energy in the economy and society. The works of Ayres and Kümmerl show that a more nuanced view of ‘energy × technology’ provides a valuable perspective for considering the role of energy technology in macroeconomic growth equations [1,2,28]. Their works counter the usual assumption of using ‘total factor productivity’ without explicitly providing an interpretation of how energy technology describes a large amount of technological growth. Stern has also demonstrated the role of long-term capital (e.g. technology) substitution for fuels and that the move toward more refined (high quality) energy carriers is causal to economic growth [40–42]. ERRs are one set of metrics for assessing ‘energy × technology’, and they provide one tool for projecting energy scenarios.

I have previously demonstrated methods for translating between ERR and monetary cost metrics. Refs. [25] and [23]
demonstrate one relatively simplified method to consider the whole system boundary of an energy business to simultaneously calculate LCOE and ERRs: all dollar expenditures require the average quantity of primary energy consumption [26], uses a price-based proxy ERR, the Energy Intensity Ratio, to show that prices (scaled and inverted to compare to ERRs) follow the same trends as separately-calculated ERRs for fossil fuels and electricity prices in the United States. Thus, energy price can be used as a proxy ERR metric. Further, King and Hall (2011) formulated a method to estimate the range of prices for U.S. oil and gas extraction based upon the EROI of oil and gas.

While [27] and [26] analyze United States data, they both demonstrate the somewhat trivial, yet often neglected point: energy cost per unit is inversely proportional to the ERR metric (e.g. $/BBL – EROI^{-1}$). The lower limit values of NER = 0 and GER = 1 correspond to infinite energy price. If GER = NER = NEER = ∞, such as for solar primary energy, the price is zero. In the real world we pay for energy commodities with NER between zero and infinity. Future research efforts should focus on refining this simple conclusion in the context the economy’s structure.

Instead of using only economic information to inform energy analyses, it is imperative that future energy thinking and scenarios use a mixture of energy and material flow information in combination with, but not 100% dependent upon, economic flow information. Otherwise, there is practically no point in performing net energy analyses for decision making because the logic is circular; translating economic information to energy units to calculate energy metrics provides no new information than using economic metrics to begin with. We know that all other things being equal, an energy resource and technology combination with a larger ERR (GER, NER, EROI, NEER, ERP, etc.) provides for a cheaper per unit energy carrier.

At some critically low aggregate ERR, a given society can no longer grow because energy prices, and thus expenditures, consume too much of disposable income. We do not exactly know the critical ERR values that indicate the need for societal changes in growth and complexity [43]. While [19] postulated the minimum EROI for liquid fuels in today’s society (to keep from shrinking or to keep growing), there is much more to fundamentally understand. ERRs measure output relative to inputs, and thus equivalent to measuring GDP (gross domestic product) relative to intermediate trade. Neither ERRs nor GDP metrics describe the internal structure of the system (economy) itself. By exploring the different internal structures we can measure and track the balance of different alternatives between efficiency and resilience for the same outputs [44]. ERRs can change (just like GDP) based upon changes in both energy extraction and energy end-use decisions and technologies (e.g., efficiency). If desired future energy carriers have lower ERRs, due to limitations in modifying either energy extraction or end-use, it is important we contemplate the necessary systemic restructuring. This paper has shown that for constant technology, system wide ERRs can change just by choosing a different mix of outputs (e.g., going to a high percentage renewable energy system from a dominant fossil energy system).

6. Conclusion

Humans did not create fossil energy just as we do not create sunlight. Using these primary energy resources in combination with conversion technologies we generate electricity for serving many energy services. It is important to remember that electricity is not a primary energy supply. Thus there is no ‘gross’ amount of electricity existing in the world for humans to extract, and the gross energy ratio, GER (=energy return on energy investment at the mine mouth, EROI_{tmm}), is not well-defined for electricity or other similar energy carriers such as gasoline and biofuels. At the full system-wide scale GER = NER + 1, and for high quality primary energy supplies at the point of extraction the difference between GER and NER is small. For smaller GER, however (<10), the difference in definitions and interpretation of ERRs, such as between NER and GER, become more important to understand.

This manuscript also indicates that even with a constant electricity generation technology assumption (here equal to constant values in technology matrix, A, for fossil and renewable electricity processes), the energy return ratios of the system-wide grid electricity change as the definition of the grid mix changes. The implication is that ERR calculations for single electricity technologies that depend on an assumed unmodeled mix of grid electricity are unable to fully inform us as to a future grid mix that is significantly different (e.g., a majority renewable electricity grid instead of a majority fossil electricity grid). The same concept holds for future scenarios with the majority of transportation fuels from renewable energy versus fossil energy. The ERRs, just like energy prices, all change simultaneously as the mix of energy consumption changes.

I encourage net energy analysis researchers to use matrix formats, as in this paper or similar constructs, to encourage better understanding and transparency in ERR calculations. I also encourage the net energy analysis community to link their calculations to economic measures, such as energy prices and expenditures, and historical trends to enhance our understanding of energy in our past, present, and future.

Appendix

A.1 Units of technological coefficient matrix A

The units of matrix A are as follows (numerator is input needed for unit output of denominator): FE = fossil energy flow in joules, Ge = grid electricity in joules, Fe = fossil electricity in joules, Re = renewable electricity in joules, M = materials in kg, RE = renewable energy flow in joules, and Se = storage of electricity in joules.

\[
A = \begin{bmatrix}
FE & FE & FE & FE & FE & FE \\
FE & Re & Ge & M & RE & Se \\
Fe & Fe & Fe & Fe & Fe & Fe \\
Fe & Ge & Re & M & RE & Se \\
Re & Re & Re & Re & Re & Re \\
Ge & Ge & Ge & Ge & Ge & Ge \\
Fe & Fe & Re & M & RE & Se \\
M & M & M & M & M & M \\
RE & RE & RE & RE & RE & RE \\
Fe & Fe & Re & M & RE & Se \\
Se & Se & Se & Se & Se & Se \\
FE & RE & Ge & M & RE & Se
\end{bmatrix}
\]  
(21)

A.2 Model results assuming no renewable electricity storage

Instead of assuming a renewable electricity storage technology that stores renewable energy to make it dispatchable, I can assume that the fossil electricity generation ramps up and down to meet the “net load = load – renewable electricity.” To some degree increased proportions of renewable electricity can cause decreases in thermal power plant efficiency, and hence system net energy. The reason for this decrease in thermal efficiency is that thermal power plants run most efficient at constant power output at the
optimal set point for torque and speed. In a previous co-authored paper, Meehan approximated that with 5–9% of annual electricity generation from wind power in the Electric Reliability Council of Texas (ERCOT) during 2008–2011 caused a few percent (~3%) higher CO2 emissions due to slightly increased ramping up and down of thermal fossil fueled generation units [30]. Other research based upon experimental conditions implies that wind power achieves approximately 75–80% of CO2 emissions reductions compared to an assumption of perfect 100% displacement of CO2 from natural gas generation [24].

For illustrative purposes of the example in this Appendix, I assume a fossil thermal power plant with a maximum efficiency of $\eta_{\text{fossil,max}} = 1/3$ with no renewable electricity on the grid (i.e., $\alpha = 1$) that linearly declines as a function of $\eta_{\text{fossil,min}} = 0.3$ with 100% renewable electricity on the grid (i.e., $\alpha = 0$). I also assume no renewable electricity storage technology. The necessary modifications to the technological coefficient matrix $A$ in Equation (14) are (1) row and column 7 of Equation (14) become all zeros (e.g., no storage technology), (2) the concept of the capacity factor, $\phi$ is no longer needed to characterize renewable electricity flow to the now non-existing storage system, and (3) and the fossil power plant efficiency, used in $A_{12}$, changes to the following:

$$ A_{12} = \frac{1}{\text{Fossil Electricity Efficiency}} = \frac{1}{\alpha \eta_{\text{fossil,max}} + (1 - \alpha) \eta_{\text{fossil,min}}} \quad (22) $$

I repeat some ERR calculations using the new assumptions of this Appendix and show the results in Table 7. For the example of a 99% renewable grid ($\alpha = 1$), 8.1 units of solar insolation and 0.19 units of fossil primary energy are required to produce the 1 unit of grid electricity at NEERsys = 0.82 compared to the modeled case in Section 4 of the manuscript that required 9.24 units of solar insolation and 0.21 units of fossil primary energy to produce a NEERsys = 0.57. The driving factors for the differences between the results of this Appendix and those of the main body of the manuscript are both lower fossil electricity conversion efficiency (due to assumed ramping effects) and higher renewable electricity conversion efficiency to the grid (due no longer including storage system losses).

Table 7

Here, system wide GEP, GERP, and NEER of calculations assuming 1 unit of grid electricity net output. This table assumes $\eta_{\text{fossil}}$ varies as in Equation (22) and $\eta_{\text{fossil,min}} = 0.15$. Because renewable energy is assumed to flow on the Earth without human intervention, GER,NER,NEER $\rightarrow \infty$ in all modeled cases.

<table>
<thead>
<tr>
<th>% Grid that is fossil electricity (a)</th>
<th>GER fossil energy</th>
<th>NER sys grid electricity</th>
<th>NEER sys grid electricity</th>
<th>Fossil energy extraction</th>
<th>Renewable energy extraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>99%</td>
<td>11.1</td>
<td>0.37</td>
<td>2.34</td>
<td>3.57</td>
<td>0.08</td>
</tr>
<tr>
<td>50%</td>
<td>7.24</td>
<td>0.20</td>
<td>1.23</td>
<td>2.00</td>
<td>4.02</td>
</tr>
<tr>
<td>10%</td>
<td>5.82</td>
<td>0.15</td>
<td>0.87</td>
<td>0.54</td>
<td>7.31</td>
</tr>
<tr>
<td>1%</td>
<td>5.59</td>
<td>0.14</td>
<td>0.82</td>
<td>0.19</td>
<td>8.06</td>
</tr>
</tbody>
</table>

References


